# Supply Management & Model Theory

**Bernard Price**
**Certified Professional Logistician**

# Operational Availability

- **The Probability that the Equipment is Operating or in a Committable Condition to Operate at any Random Point in Time**
- **Quantitative Expression of User Need**

# Derivation of Operational Availability

$$Ao = \frac{\text{UP TIME}}{\text{UP TIME + DOWN TIME}} = \frac{\text{UP TIME / SYSTEM FAILURE}}{\text{(UP TIME + DOWN TIME) / SYSTEM FAILURE}}$$

$$\text{UP TIME / SYSTEM FAILURE} = MCTBE_{SYSTEM}$$

$$\text{DOWN TIME / SYSTEM FAILURE} = MSRT_{SYSTEM}$$

$MCTBE_{SYSTEM}$ - Mean Calendar Time Between System Failure

$MSRT_{SYSTEM}$ - Mean System Restoral Time per System Failure

# Mean System Restoral Time Breakdown in Models

$$MSRT_{SYSTEM} = MTR_{SYSTEM} + CWT_{SYSTEM}$$

**MTR - Mean Time to Restore with 100% Stock Availability Forward**

**CWT- Mean Customer Wait Time at Forward Level per System Failure**

$$MTR_{SYSTEM} = MTTR_{SYSTEM} + MRDT_{SYSTEM}$$

**MTTR – Mean Time to Repair when all Resources with**

# Restoral Delay Time Contributors

- **Spares are Not Collocated with Equipment**
  - **Spares are Delivered Forward to Restore**
  - **Contact Maintenance Team Restores Equipment**
  - **Equipment is Evacuated to Restore**
- **Some ILS Elements May Not Be Satisfactory**
  - **Personnel Lacking Appropriate Skills**
  - **Personnel Not Available**
  - **Non-Functioning TMDE Forward**
  - **Forward Repair Documentation Insufficient**

# Relationship of Customer Wait Time (CWT) to Logistics Response Time

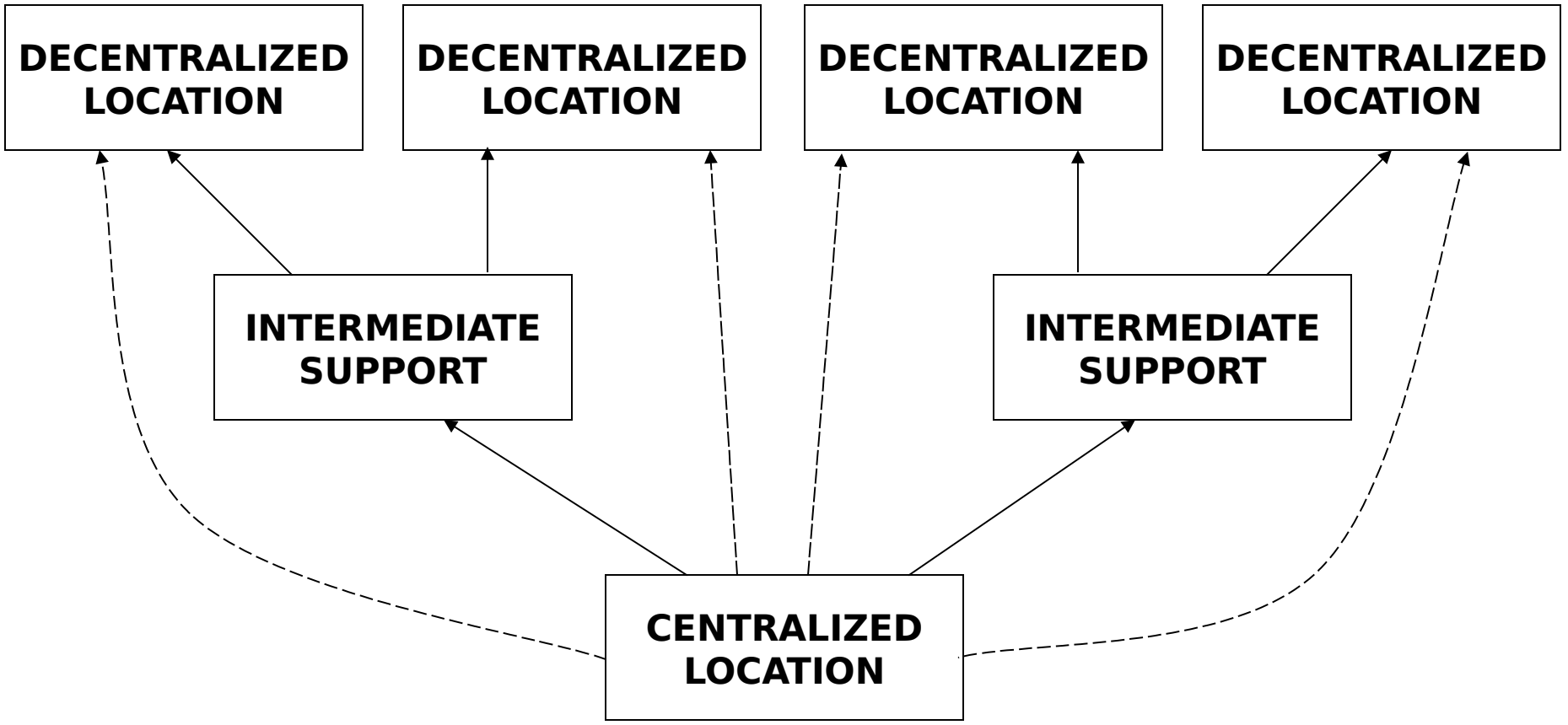
$CWT = SA_1 \times 0 + (1 - SA_1) \times MTTO_1$

$CWT = (1 - SA_1) \times MTTO_1$

$SA_1$ - Stock Availability at (or Probability of Filling an Order from) Forward Level Stock

$MTTO_1$ - Mean Time to Obtain a Line Replaceable Unit (LRU) Spare at Forward Level Support

# Traditional Supply Flow



# Inventory Distribution

| STOCKAGE LOCATION | COMMERCIAL | GOVERNMENT |
| --- | --- | --- |
| CENTRALIZED LOCATION | MANUFACTURER OR PLANT WAREHOUSE | DEPOT SUPPORT OR WHOLESALE LEVEL |
| INTERMEDIATE SUPPORTS | DISTRIBUTION CENTER OR REGIONAL WAREHOUSE | GENERAL SUPPORT DIRECT SUPPORT (AUTHORIZED STOCKAGE LIST) |
| DECENTRALIZED LOCATION | RETAIL STORE OR CUSTOMER | ORGANIZATIONAL SUPPORT OR SITE (PRESCRIBED LOAD LIST) |

# Influence of Maintenance on Supply Support

- **MTTO, or the logistic pipeline time for an item, is dependent on the item's maintenance policy as well as its supply time**

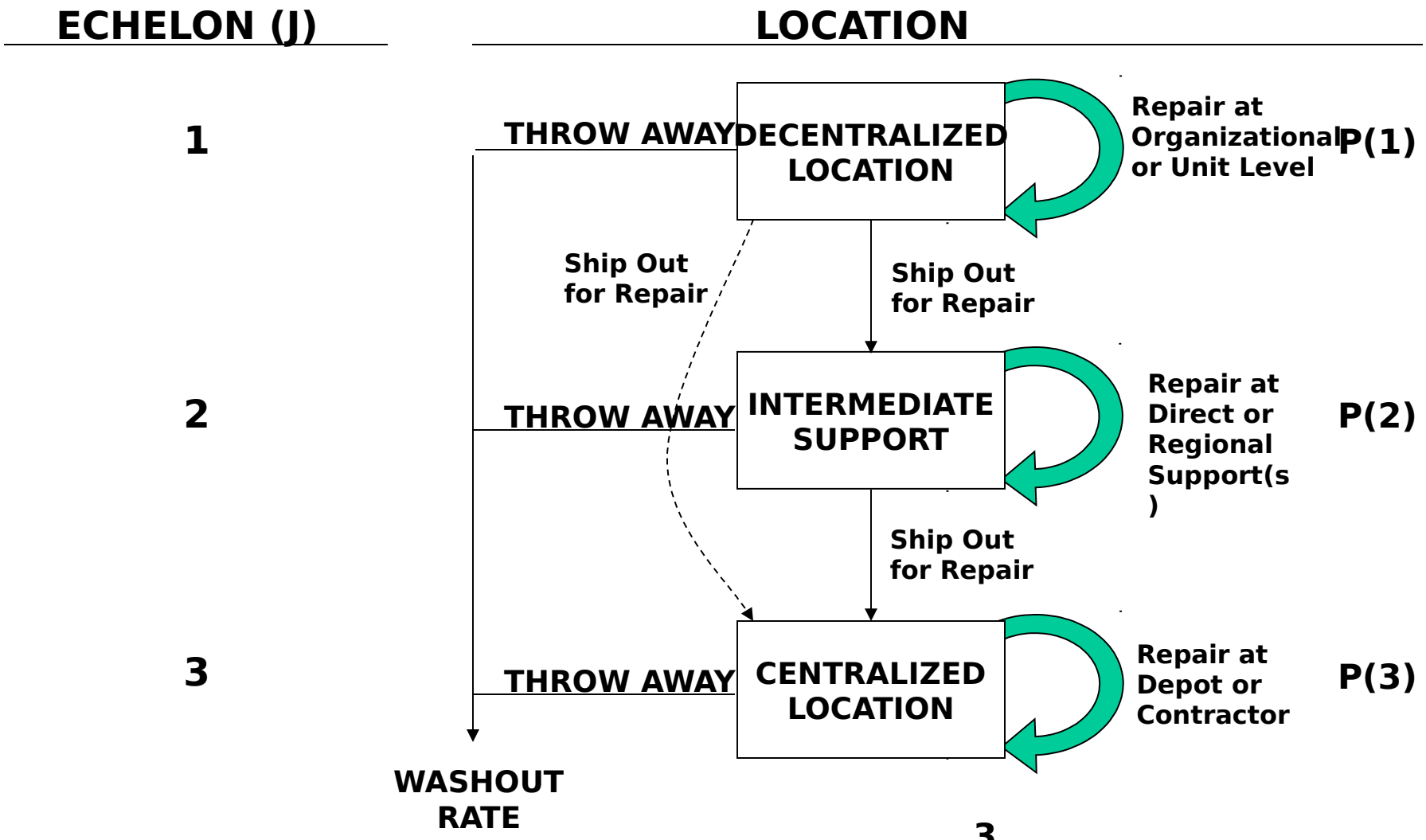
- **A removal and replacement maintenance action of an item causes a demand for a spare to occur within the supply system**

- **A successful item repair causes that item to become operable again and placed either into stock or sent back to the customer**

- **An unsuccessful repair action or throwaway of an item requires reprocurement of that item if the stock levels are to be replenished**

- **The Unserviceable Return Rate of reparable items impacts reprocurement or repair because an item not**

# Maintenance Distribution




NOTE: P(J) is percentage of repairs made at echelon J

$$\sum_{J=1}^{3} P(J) +$$

Washout Rate = 1

# Logistics Response Time Terms Driving MTTO

## Order & Ship Time ($OST_j$):

Total time for a lower echelon stock point j to order and receive a shipped spare or component from a higher echelon (more centralized) stock point

## Repair Cycle Time ($RCT_{i,j}$):

Total time from failure occurrence to repair of Item i at maintenance support echelon j until Item i is properly placed

- For Removal & Replacement Action, RCT includes all wait times, shipment time to echelon j & repair turnaround time at echelon j & placement of repaired item into stock at echelon j.

- For Repair & Return Action, RCT includes all wait times, shipment times to & from echelon j & repair turnaround time at echelon j until placement of repaired item back into equipment

# Other Terms Driving the MTTO Spares

## Maintenance Task Distribution ($MTD_{i,j}$):

$$\sum_{J=1}^{3} P_{i,j} + RR_i = 100\%$$

$P_{i,j}$ Probability or Percentage of time Item i is repaired at maintenance support echelon j.

.

**Replacement Rate ($RR_i$):** Probability Item i is replaced

## Mean Time to Obtain a Back Order ($MTTOBO_{i,3}$):

Average time to obtain Item i at the wholesale/depot support level when a back order has occurred. (accounts for previous orders or repairs of Item i due in )

## Back Order Rate ($1 - SA_{i,3}$):

Probability or Percentage of time an order for Item i is not in stock at the wholesale/depot support level

# Mean Time To Obtain (MTTO) Spares at Retail Support Levels

- $MTTO_1 = RCT_1 \times P_1 + (1 - P_1) \times (OST_{2_1} + (1 - SA_2) \times MTTO_2)$
- FOR LRU THROWAWAY OR REPAIR AT CENTRALIZED LOCATION

$MTTO_2 = OST_{3_2} + (1 - SA_3) \times MTTOBO_3$

- FOR LRU REPAIR AT THE INTERMEDIATE LOCATION

$TTO_2 = RCT_2 \times P_2 + (1 - P_2) \times (OST_{3_2} + (1 - SA_3) \times MTTOBO_3)$

OST - ORDER & SHIP TIME
SA - STOCK AVAILABILITY (ORDER FILL RATE)
P - PERCENTAGE REPAIRED AT ECHELON
RCT - REPAIR CYCLE TIME

# System Mission Success

**When failures occur randomly yielding a constant failure rate, the System Reliability or the Probability of mission success without a system failure is exponentially distributed**

**For an Exponential Distribution:**

$$P(0) = e^{-\lambda t}$$

**Where:**

- ***P*(0) is the probability of having no failures over a mission time period**
- **λ is the failure rate (failure/ hour) for one system**
- ***t* is the length of mission time**

**∴ λ*t* is the average number of system failures occurring over the length of mission time**

# Mission Success With Multiple Items

**Given that failures occur randomly and are exponentially distributed, item success predictions are based on the Poisson Distribution**

**For a Poisson Distribution:**

$$P(x) = \frac{n\lambda t^{x} \cdot e^{-n\lambda t}}{x!}$$

**Where:**

- ***P*(x) is the probability of having x failures over the mission time**
- ***n* is the number of items operating**
- **λ is the failure rate (failure/ hour) for one item**
- ***t* is the length of mission time**

**∴ *n*λ*t* is the expected number of failures occurring over the length of mission time**

# Working with Poisson Distribution

$$P(x)=\frac{n\lambda t^{x}\cdot e^{-n\lambda t}}{x!}$$

$$P(0)=e^{-n\lambda t}$$

Note: $n\lambda t=-\ln P(0$

$$P(1)=n\lambda t\cdot e^{-n\lambda t}=n\lambda t\times P(0$$

$$P(2)=\frac{n\lambda t^{2}}{2}\cdot e^{-n\lambda t}=\frac{n\lambda t}{2}\left[n\lambda t\cdot e^{-n\lambda t}\right]=\frac{n\lambda t}{2}\times P(1)$$

$$P(3)=\frac{n\lambda t^{3}}{6}\cdot e^{-n\lambda t}=\frac{n\lambda t}{3}\left[\frac{n\lambda t^{2}}{2}\cdot e^{-n\lambda t}\right]=\frac{n\lambda t}{3}\times P(2$$

**If the mission reliability is stated, the expected number of failures occurring over the length of mission time can be computed.**

- ***P*(0) is the mission reliability for all items not failing during the mission**
- ***P*(1) is the probability of having 1 failure during the mission time**
- ***P*(0) + *P*(1) is the probability of mission success with 1 or less failures during the mission time**

**Poisson Distribution generalization:**

$$P(x)=\frac{n\lambda t}{x}\cdot P(x-1)$$

# Example:

- Suppose an item has a mission reliability of 60.65%. How many items must be used to have at least a 99% chance of succeeding the mission?

P(0) = 0.6065

$$n\lambda t = -\ln P(0) = -\ln 0.6065 = 0.50$$

0.5 represents the expected number of failures during the mission duration

$$P(1) = 0.5 \times P(0) = 0.5 \times 0.6065 = 0.3033$$

Having 1 additional item to accomplish the mission increases the probability of mission success to 0.6056 + 0.3033 = 90.98%

## Example (cont'):

$$P(2) = \frac{0.5}{2} \times P(1) = \frac{0.5}{2} \times 0.3033 = 0.075$$

The Δ improvements to mission success by having a second additional item available to accomplish the mission is 7.58%. The probability of missions success with 3 items is:

$$0.9098 + 0.075 = 0.9856 \approx 0.9$$

$$P(3) = \frac{0.5}{3} \times P(2) = \frac{0.5}{3} \times 0.075 = 0.012$$

Having 3 additional items to accomplish the mission increases the probability of mission success to $0.9856 + 0.012 = 0.9976 \approx 99\%$

- Therefore, 4 items must be used to have at least a 99% chance of succeeding the mission

# Stock Availability / Order Fill Rate Predictions

**Order Fill Rate:**

- **The probability of filling orders for an item before replenishment spares are obtained**
- **It is assumed that retail level uses an existing spare parts if available, and orders a replacement spare simultaneously**
- **At retail levels, the order quantity is assumed to be 1 (q=1)**
- **Without out a spare the probability of filling an order before replenishment is 0%**

With 1 spare on hand initially the probability of filling all orders prior to replenishment is the same as having a mission success with 0 spares available initially and a mission time equal to the Mean Time To Obtain (MTTO) a spare. (i.e. Fill Rate = $e^{-mt}$)

# Stock Availability / Order Fill Rate From Previous Example

**With t = MTTO:**

***nλt* is the expected number of demands occurring over the average replenishment time to obtain a spare**

**Example Results:**

- **With 0 spares, SA = 0% as there is no stock to fill orders**
- **With 1 spare, SA = 60.65% order fill rate**
- **With 2 spares, SA = 90.98% order fill rate**
- **With 3 spares, SA = 98.56% order fill rate**
- **With 4 spares, SA = 99.82% order fill rate**

# Optimizing Sparing Costs

- **Trade-off analysis is necessary which varies Retail Level supply stock availabilities to yield varied mixes of PLL and ASL LRU stockage quantity**

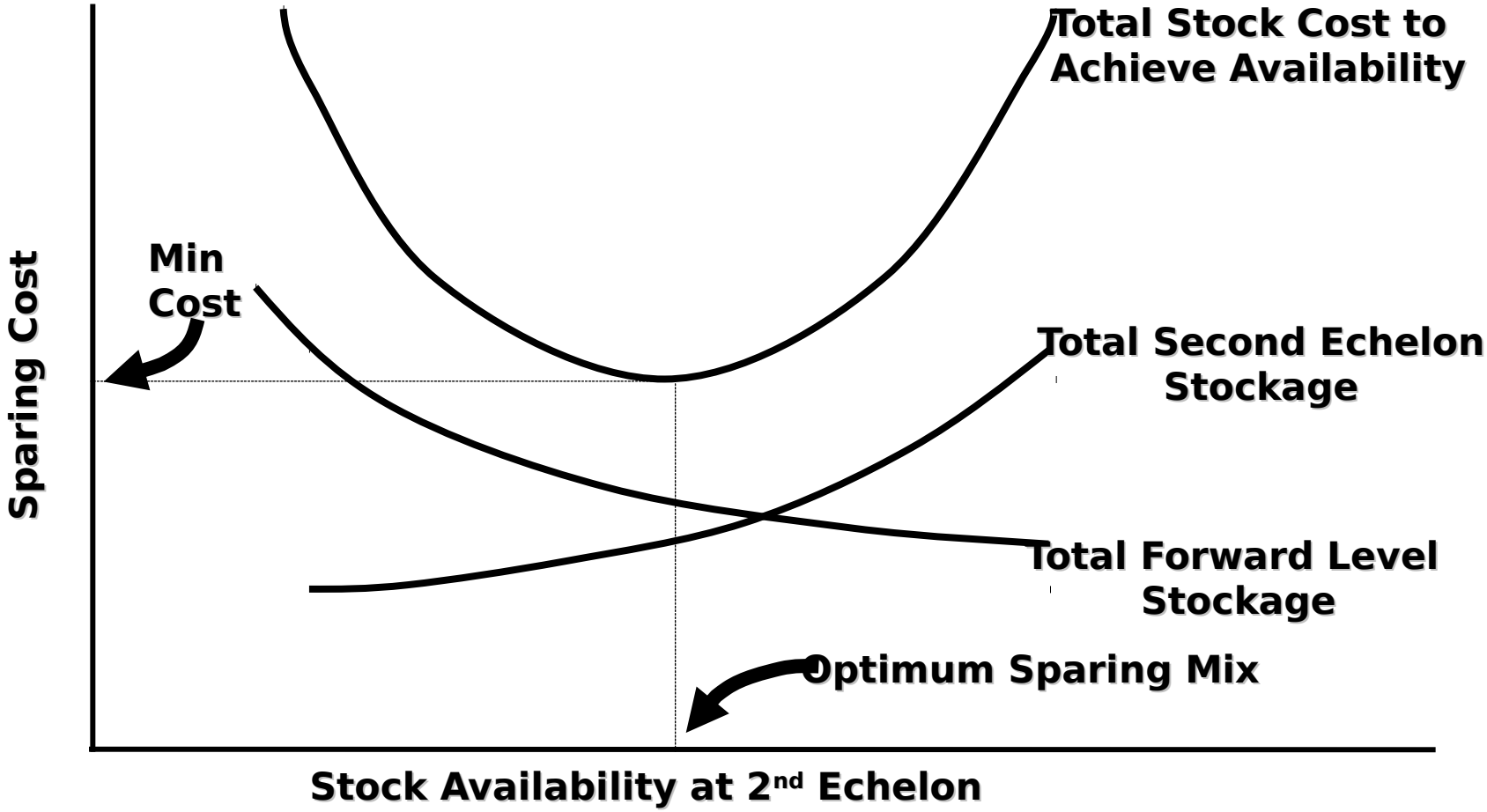
- **Increasing ASL order fill rates produces more ASL LRU spare quantities and less PLL Customer Wait Time**

- **The higher cost of sparing more at the centralized location reduces the stock needed at the decentralized locations**

- **The cost savings per decentralized location is magnified by the number of locations being supported by a centralized location**

- **Decreasing ASL order fill rates produces less ASL LRU spares and more PLL Customer Wait Time. More PLL LRU spares are needed to achieve the same availability goal**

# Multi-Echelon Sparing Optimization to Same Equipment Availability



# Selected Essential-item Stock to Availability Method (SESAME) Usefulness

- **Optimizes Multi-Echelon Retail Level Initial Sparing to Achieve End Item Ao Requirement or Forward Level Support Stock Availability Goal**

***-OR-***

- **Evaluates Ao or SA Based on Sparing Mix, LRU Reliabilities and Logistics Response Times**

***-OR-***

- **Optimizes Plus Up Sparing to Achieve Ao or SA Goal Given the Present Retail Level Sparing Mix**

**Maintenance Concept for each Essential Item is Proposed or Known**